Dynamics of Bitcoin Mining

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Dynamics of Bitcoin Mining

What happens to mining when

- the Bitcoin price changes,
- there are mining supply shocks,
- the price of energy changes,
- hardware technology evolves?

Fundamental Equation of Mining (per block)

$$p_b(R + F) \geq p_e E + NC\tau$$

- p_b: price of Bitcoin ($/BTC)
- E: energy used (kWh/block)
- R: reward (BTC/block)
- F: fees (BTC/block)
- p_e: price of electricity ($/kWh)
- N: number of miners
- C: cost of hardware (per unit of time)
- τ: block time

Fundamental Equation of Mining (per hash)

$$V_h \geq p_e \alpha + p_h$$

- V_h: \$/hash
- p_e: price of electricity (\$/kWh)
- α : efficiency (kWh/hash)
- p_h: hardware cost (\$/hash)

Same equation but divided by H (global hash/s) and τ (s/block)

When Bitcoin price goes up

$$V_h \geq p_e \alpha + p_h$$

- V_h goes up, profits increase
- More miners come in, H goes up
- V_h comes back down
- Total computation increases in proportion to the value of the network but
- V_h the $/hash is automatically stabilized! It doesn't depend on $/BTC.

When there's a supply shock

$$V_h \geq p_e \alpha + p_h$$

- E.g. one government bans mining suddenly: H goes down
- τ block time goes up but V_h stays the same
- Difficulty adjusted downwards
- V_h goes up and profitability increases for surviving miners
- New miners, previously kept out by higher cost ($p_e \alpha + p_h$), come in
- V_h stabilizes at a new higher value than before

Energy market changes

$$V_h \geq p_e \alpha + p_h$$

- Miner with lower p_e than average will grow their hashrate
- H increases, difficulty gets adjusted up
- V_h decreases
- Higher cost miners become unprofitable and drop out
- V_h stabilizes at a new lower value
- Continuous migration toward the lowest cost of electricity worldwide

Hardware market changes

$$V_h \geq p_e \alpha + p_h$$

- Miner with lower p_h or lower α than average will grow their hashrate
- H increases, difficulty gets adjusted up
- V_h decreases
- Higher cost miners become unprofitable and drop out
- V_h stabilizes at a new lower value
- Continuous migration toward the cheaper and more efficient hardware

Halving

$$p_b(R + F) \geq p_e E + NC\tau$$

$$V_h \geq p_e \alpha + p_h$$

- R goes down by ½, V_h goes down 40%–50%, profits decrease
- Miners whose gross margin is below that get pushed out
- H goes down, τ block time goes up, difficulty adjusts down,
- V_h goes back up but stabilizes at new lower value than before
- MAYBE market price p_b increases (if demand is fixed, but new supply flow decreases)
- H to rise again but V_h the $/hash is stable at new lower value

Transaction Fees

$$p_b(R + F) \geq p_e E + NC\tau$$

$$V_h \geq p_e \alpha + p_h$$

- 1MB/10min = 1.67 kB/s of bandwidth for transactions
  - Segwit Bytes -> vBytes
- Still, will always have < 10 transactions/second
- Fees are congestion price for bandwidth
- Lower value transactions migrate to layer 2 (e.g. Lightning)
- Boundary is dynamic based on adoption and demand for transactions
- Equilibrium, like “Paris Metro Pricing”

Computing efficiency

$$p_b(R + F) \geq p_e E + NC\tau$$

$$V_h \geq p_e \alpha + p_h$$

- Landauer's principle states that each bit of output must consume kT ln(2) Joules
- So theoretical minimum is $\alpha \approx 2 \times 10^{-13}$ kW h/TH
- If Moore's law applies, we get to the limit in about 40 years
- 100M more efficient than today but never zero energy